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## Numerical Methods - MA 207 <br> Numerical Integration

1. $I=\int_{1}^{3} \frac{d x}{x}$ is evlauated by trapezoidal rule with 8 strips. Estimate the error in the value of $I$.
2. Evaluate

$$
\int_{0}^{6} \frac{d x}{1+x^{2}}
$$

by using
(a) Trapezoidal rule
(b) Simpson's $1 / 3$-rule
(c) Simpson's 3/8-rule Weddle's rule
and compare the results with its actual value.
3. Evaluate

$$
\int_{0}^{1} \frac{x^{2}}{1+x^{2}} d x
$$

by using Simpson's 1/3- rule. Compare the error with the exact value.
4. Use the Trapezoidal rule to estimate the integral

$$
\int_{0}^{2} e^{x^{2}} d x
$$

taking 10 sub-intervals.
5. Use Simpson's 1/3-rule to find

$$
\int_{0}^{0.6} e^{-x^{2}} d x
$$

by taking seven ordinates. Compare the approximate with the exact value.
6. Using Simpson's 3/8-th rule, compute the value of

$$
\int_{0.2}^{1.4}\left(\sin x-\log x+e^{x}\right) d x .
$$

7. The velocity $v(\mathrm{~km} / \mathrm{min})$ of a moped which starts from rest, in given at fixed intervals of time $t$ (min)
as follows $\left.\begin{array}{ccccccccccc}t: & 2 & 4 & 6 & 8 & 10 & 12 & 14 & 16 & 18 & 20 \\ & v: & 10 & 18 & 25 & 29 & 32 & 20 & 11 & 5 & 2\end{array}\right)$

Estimate approximately the distance covered in 20 minutes.
8. The velocity $v$ of a particle at distance $s$ from a point on its linear path is given by the following table:

$$
\begin{array}{rccccccccc}
s(m): & 0 & 2.5 & 5.0 & 7.5 & 10 & 12.5 & 15 & 17.5 & 20 \\
v(\mathrm{~m} / \mathrm{sec}): & 16 & 19 & 21 & 22 & 20 & 17 & 13 & 17 & 9
\end{array}
$$

Estimate the time taken by the particle to traverse the distance of 20 meters, using Boole's value.
9. A solid of revolution is formed by rotating about the $x$ - axis, the area between the $x$ - axis, the lines $x=0$ and $x=1$ and a curve through the points with the following co-ordinates.

| $x:$ | 0 | 0.25 | 0.5 | 0.75 | 1 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $y:$ | 1 | 0.9896 | 0.9589 | 0.9089 | 0.8415 |

Estimate the volume of the solid formed using Simpson's rule.
10. A river is 80 ft . wide. The depth $d$ in feet at a distance $x \mathrm{ft}$. from one bank is given by the following table. Find approximately the area of the cross-section.

| $x:$ | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $y:$ | 0 | 4 | 7 | 9 | 12 | 15 | 14 | 8 | 3 |

11. A body is in the form of a solid of revolution. The diameter $D$ is cm . of its sections at distances $x \mathrm{~cm}$. from on end are given below. Estimate the volume of the solid.

$$
\begin{array}{llllllll}
x: & 0 & 2.5 & 5 & 7.5 & 10 & 12.5 & 15 \\
D: & 5 & 5.5 & 6 & 6.75 & 6.25 & 5.5 & 4
\end{array}
$$

12. A rocket is launched from the ground. Its acceleration is registered during the first 80 seconds and is given in the table below. Using Simpson's 1/3-rd rule, find the velocity of the rocket at $t=80$ seconds.

$$
\begin{array}{rlllllllll}
t(\mathrm{sec}): & 6 & 10 & 20 & 30 & 40 & 50 & 60 & 70 & 80 \\
f\left(\mathrm{~cm} / \mathrm{sec}^{2}\right) & 30 & 31.63 & 33.34 & 35.47 & 37.75 & 40.33 & 43.23 & 46.69 & 50.67
\end{array}
$$

13. Derive composite Simpson's $1 / 3$-rule.
14. Derive composite Simpson's 3/8-rule.
15. Using composite Trapezoidal rule, evaluate

$$
\mathrm{I}=\int_{1}^{2} \int_{1}^{2} \frac{d x d y}{x y}
$$

taking four subintervals in each direction.
16. Apply composite Simpson's $1 / 3$-rule to evaluate the integral

$$
\mathrm{I}=\int_{0}^{1} \int_{0}^{1} x e^{y} d x d y,(h=k=0.5)
$$

17. Evaluate $\int_{0}^{1} \int_{0}^{1}(x+y) d x d y$ using Simpson's $1 / 3$ rule with $h=k=0.5$.
